Knowability and Singular Thought: *De Re* Knowledge and Semantic Ascent

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In 1963 Frederich Fitch published a paper titled "A Logical Analysis of Some Value Concepts".[1] In this paper he introduced six theorems, of which little was discussed. The significance of these theorems, particularly, Theorem 4, credited to an anonymous referee–now known to be Alonzo Church[2]–, and Theorem 5, a generalization on Theorem 4, was not noticed until thirteen years after the original publication.[3] Hart and McGinn (1976) pointed out that Theorem 5 collapses a weak epistemic principle commonly attributed to verificationists or anti-realists into a position of naive idealism: all truths are known by some person at some time. It is the purpose of this paper to examine theorems 4 and 5, as it has been noted[4] that these theorems establish necessary limits on one's epistemic ability independently of any considerations related to the commitments of the anti-realist, and to then show that the only way in which this collapse can be generated is through singular thought about abstract objects. The argument will proceed in two steps: in §1, I outline the modal and epistemic principles required in order to generate Theorems 4 and 5, and then show how these theorems, in conjunction with the weak verificationist principle, generate the modal-epistemic collapse; in §2, I argue that the only possible reading of the fitch proposition which generates the collapse requires singular thought about propositions, and I then examine one way in which singular thought about a proposition might make sense.

[1] Fitch (1963), Reprinted Salerno ed. (2009b).
[2] Cf. Salerno (2009a).
[3] Cf. W.D. Hart and Colin McGinn (1976).
[4] Williamson (2000), Kvanvig (2006).

1 The Church-Fitch Paradox of Knowability

Theorems 4 and 5, as originally provided by Fitch, set certain limits on our epistemic abilities. There exist truths that are necessarily unknowable. This observation was first made in 1945 by Alonzo Church, in a referee report sent to the Journal of Symbolic Logic[5], where he provided trivialization results for an earlier paper by Frederic Fitch, "A Definition of Value," which was not published. Church's theorem, Theorem 4, and Fitch's generalization, Theorem 5, are as follows:[6]

Theorem 4. $\Vdash \exists s \exists t \exists \rho(\rho \wedge \neg sK_t\rho) \supset \exists s \exists t \exists \rho(\rho \wedge \neg\Diamond sK_t\rho)$

Theorem 5. $\Vdash \forall s \forall t \exists \rho(\rho \wedge \neg sK_t\rho) \supset \forall s \forall t \exists \rho(\rho \wedge \neg\Diamond sK_t\rho)$

The two theorems above have slightly different implications, but they are both generated by the same *reductio* argument. In what follows I first outline the inference rules necessary for the *reductio*, develop the argument, and then briefly interpret the general results.

1.1 The *Reductio*, General Results

The *reductio* argument requires two epistemic principles and two modal principles:

KIT. $Kp \Rightarrow p$

K-DIST. $K(p \wedge q) \Leftrightarrow Kp \wedge Kq$

NEC. From $\Vdash p$, infer $\Vdash \Box p$

M-EQ. $\Box\neg p \Leftrightarrow \neg\Diamond p$

The epistemic pair, KIT and K-DIST, correspond to the factivity of knowledge, that knowledge implies truth, and that knowledge distributes over conjunction, one of the weakest forms of epistemic closure. The modal pair corresponds to the necessitation of theorems and the interdefinability of the modal operators.

[5] Cf. Church (2009), Salerno (2009a)

[6] Note that these theorems include second-order quantification over propositions. In order to not confuse an instance of a particular proposition with a bound second-order variable, I will use '$\rho_1, \rho_2, \ldots$' to represent second-order variables ranging over the domain of propositions, suppressing subscripts when possible, and '$p, q, r, \ldots$' to represent particular propositions when instantiated.

The *reductio* itself is generated through the assumption that an unknown truth–the *fitch proposition*, $p \wedge \neg Kp$–is known, which generates a contradiction given KIT and K-DIST, and, thus, we have:

$$\Vdash \neg K(p \wedge \neg Kp)$$

We then apply NEC:

$$\Vdash \Box\neg K(p \wedge \neg Kp)$$

And, by M-EQ, we have that it is not possible to know the fitch proposition:

$$\Vdash \neg\Diamond K(p \wedge \neg Kp)$$

It is worth noting that the theorem resulting from the *reductio* argument differs from Theorems 4 and 5, but is necessary for both. We can further generalize, since the proposition assumed is arbitrary, and we have been suppressing (arbitrary) subjects and times:

Theorem FP. $\Vdash \forall s \forall t \forall \rho \neg\Diamond s K_t(\rho \wedge \neg s K_t \rho)$

We now have two general theorems, theorems 5 and FP, which serve to bound our epistemic reach.

Theorem 5 states that there exist particular truths that no one can possibly know, while theorem FP states that for every person, indexed to particular times, there are many truths of a particular form that it is not possible to know. We will take each in turn. First, theorem 5, that which has traditionally been associated with the paradox, states that if propositions which take the form of the fitch proposition exist, then for any subject and time, there exist propositions that are not possible to know; namely, those just mentioned. Unknown truths do exist, and, if they exist, then the fact that they are unknown and true is unknowable.[7] Theorem FP, then, gives us an idea of what they look like. To be precise, it is not possible for some subject S to know at time t the proposition expressed by p being both true at t and unknown by S at t. Now, it is important to note here that what is impossible to know is not p in these cases, but the conjunction of p and the fact that it is not known, by the particular subject in question, at the particular time in question, that p. To highlight this, suppose that some subject S_1 does not know the birthday of her friend S_2's grandmother. Now, S_2 knows her own grandmother's birthday, and S_2 could easily convey this information to S_1. But at this particular point in

[7] Cf. Williamson (2000) p. 272-273.

time, t_1, the proposition expressed by the fact that S_1 does not know S_2's grandmother's birthday at t_1, conjoined with the proposition *that ϕ is s_2's grandmother's birthday* is necessarily unknowable by S_1. At some future point in time, t_2, S_1 could come to know S_2's grandmother's birthday, and S_1 could even come to know that S_1 did not know S_2's grandmother's birthday at t_1. But *at* t_1, this complex proposition was necessarily unknowable *by* S_1. In fact, at t_1, there would be no problem if S_2 came to know this complex proposition about S_1. This is just to say that the necessarily unknowable propositions characterized by Theorem FP are relativised to speakers and times. So, for any particular time t', there will, for each speaker S_i, be a fixed class of propositions necessarily unknowable by S_i. At each particular point in time, the class itself is characterized by all of those truths that the speaker does not know, conjoined with the proposition expressed by the fact that the speaker does not know them at that time. But this class shifts over time. For at $t'' > t'$, the class $UK_{s_i}^{t'}$ becomes knowable by S_i, and some new class $UK_{s_i}^{t''}$ becomes impossible to know.[8]

1.2 The Collapse

Now that we have the general results in hand, it is time to look at the modal collapse. The collapse itself is generated from the contraposition of Theorem 5:

Theorem 5^c. $\Vdash \exists s \exists t \forall \rho(\rho \supset \Diamond s K_t \rho) \supset \exists s \exists t \forall \rho(\rho \supset s K_t \rho)$

Theorem 5^c itself looks like a modal collapse. But as we have just seen, there do indeed exist truths which are necessarily unknown, so the contrapositive here must have a false antecedent. That is to say, if the relevant candidate substitution instance of ρ is false, then we have a vacuous antecedent and a vacuous consequent. If, on the other hand, we are looking at true proposition, Theorem FP shows that the consequent of the conditional in the antecedent of Theorem 5^c is false, and therefore the antecedent itself is false, given that the embedded conditional has a true antecedent and a false consequent. So Theorem 5^c is true, but vacuous. This makes trouble for the verificationist, however. For, historically, the antecedent of this conditional has been attributed to the verificationist as the principle that truth is weakly epistemically connected:[9]

WVER. $\exists s \exists t \forall \rho(\rho \supset \Diamond s K_t \rho)$

[8] Cf. Routley (1981)

[9] Cf. Salerno (2009a) for a comprehensive list of historical figures which might be considered committed to this position. To my knowledge, Dummett is the only figure to actually assent to this attribution. Cf. Dummett (2009), but given his interpretation of intuitionistic logic, he argues that the collapse is not critical for his system.

Thus, the verificationist is committed to the *truth* of the antecedent. Through Theorem FP, WVER can be shown to collapse into the absurd conclusion that is the consequent of Theorem 5^c–that all truths are already known. This collapse is induced by instancing ρ in the above principle as the fitch proposition:

$$(p \wedge \neg Kp) \supset \Diamond K(p \wedge \neg Kp)$$

But Theorem FP above shows that the consequent of this conditional is false, so the antecedent must be false, which, by trivial logic, is just an instance of the consequent of Theorem 5^c, and since p has been arbitrary throughout, we can generalize. This consequence of WVER has become known as strong verificationism, or naive idealism:

SVER. $\exists s \exists t \forall \rho(\rho \supset sK_t\rho)$

At this point, the defenders of WVER are taken to be refuted, given that their position that truth is weakly epistemically connected collapses into the absurd position that all truths are already known.

As was stated previously, it is my position that this collapse only affects those committed to the truth of the antecedent in Theorem 5^c. For the rest of us Theorem FP shows that the antecedent is false and thus the conditional itself is vacuous. There has been a recent challenge,[10] however, to the effect that the modal collapse in Theorem 5^c needs explanation prior to any commitment to WVER. This challenge has been taken up by Jenkins (2009), and, before continuing, I will briefly provide some additional reasons for thinking that this collapse is peculiar to just those who are committed to the truth of the antecedent. Jenkins considers this problem the 'New Puzzle', and characterizes it with the following questions:[11]

- Does this apparent modal collapse really occur?
- If it does not, where does the Church-Fitch proof go wrong?
- If it does, what satisfying explanation can we give of this collapse?

The way these questions are posed, they are intended to target those not explicitly committed to the truth of WVER. But as stated earlier, it would seem that if the collapse occurs, it occurs vacuously. A real problem only arises when one is committed to the *truth* of the antecedent, such as those committed to WVER. For then, by *modus ponens*, we can discharge the consequent of Theorem 5^c,

[10] Cf. Kvanvig (2006, 2009)
[11] Jenkins (2009), p. 304

which gives us the absurd conclusion that all truths are known. Without this move, we have just as much need to worry about SVER as we do about 'the moon is made of green cheese' in the conditional 'If 2+2=5, the moon is made of green cheese'.

The final argumentative route that has been taken is to couch this purported worry in terms of disappearing operators. It is simply a general concern that the modal operator disappears when moving from the consequent of the embedded conditional in the antecedent of Theorem 5^c to the consequent of the embedded conditional in the consequent of Theorem 5^c. Jenkins (2009) has provided several cases of collapses in the consequents of conditionals that are unproblematic, but in order to drive this point home, here are two additional examples where the antecedent conditional is fixed to that of Theorem 5^c:

1. $\Vdash \exists s \exists t \forall \rho(\rho \supset \Diamond s K_t \rho) \supset \forall \rho(\rho \supset \rho)$
2. $\Vdash \exists s \exists t \forall \rho(\rho \supset \Diamond s K_t \rho) \supset \exists s \exists t \forall \rho(\rho \supset \neg s K_t \neg \rho)$

Neither of these results seem *prima facie* problematic, but in both cases we have the disappearance of a modal operator. The first case simply shows that contradictions are false, the second supports the factivity of knowledge. Thus, it would seem that if there is a problem with the Church-Fitch proof, it would have to be in combination with the acceptance of WVER, not with the general theorems involved.

2 *De Re* Knowledge and Singular Thought

In this section I first outline some standard examples of *de re/de dicto* ambiguity which serve to show that when a propositional attitude report's complement clause contains a variable bound from the outside by an existential quantifier the entertainment of a singular thought about the object quantified over is required for the truth of the report in question. Second, I show that it is exactly this exported context that is required to generate the contradiction upon which Theorem FP relies. And, finally, I show that attempts to generate the contradiction through the other possible readings fail.

To begin, it must be noted that Theorem FP as it is currently formulated is necessitated prior to generalization. This is not required; one may generalize prior to necessitation with the following result:[12]

Theorem FP'. $\Vdash \Box \forall \rho \neg K(\rho \wedge \neg K \rho)$

[12] I will be suppressing quantification over subjects and times for expository purposes.

Now, we can formulate Theorem FP' with the negation taking wide scope if we so choose, through DeMorgan's for the quantifiers and M-EQ:

Theorem FP". $\Vdash \neg\Diamond\exists\rho K(\rho \wedge \neg K\rho)$

And this just goes to show that there are two equivalent methods of deriving the result. We may begin the *reductio* argument highlighted earlier with an arbitrary instance, ending with $\forall I$, or as an existential generalization, ending with $\neg I$. This equivalence is important, since the second method–where there is an addition of a wide scope existential quantifier to the *reductio* assumption–highlights the features that follow.

Now, consider the following:[13]

(1) Mary believes that a friend of mine is a bus driver.

This sentence is ambiguous between two distinct readings:

(2a) Mary believes that there is a friend of mine who is a bus driver.

(2b) There is a friend of mine whom Mary believes to be a bus driver.

The sentence (2a) requires only that Mary believe a general proposition, that I have a friend who is a bus driver. The (2b) reading, however, requires Mary to entertain a singular thought about a particular friend of mine. This requirement is termed ANTI-LATITUDE by Manley and Hawthorne (*m.s.*):[14]

ANTI-LATITUDE. Any attitude report whose complement clause contains either a singular term or a variable bound from outside by an existential quantifier requires for its truth that the subject have a singular thought.

This difference between the (2a) and (2b) reading is generated by an ambiguity related to whether the propositional attitude verb takes scope over the existential quantifier, or *vice verse*:

(3a) Mary believes $(\exists x)$ (x is a friend of mine and x is a bus driver)

(3b) $(\exists x)$ (x is a friend of mine and Mary believes that x is a bus driver)

It should be noted, however, that there is a third, intermediate, reading of (1) that will be important in what is to come:

(1*) $(\exists x)$ Mary believes (x is a friend of mine and x is a bus driver)

[13] Fodor (1970), p. 6.

[14] Manley and Hawthorne, *The Reference Book.* Manuscript. p. 29. Additionally, Cf. Sosa (1970) and Chisholm (1976).

This intermediate reading corresponds to Mary having a singular thought about a particular individual such that she believes that that person is both a friend of mine and a bus driver. Thus, on this reading, I could have a friend who is a bus driver, which would render (3a) true, but in the case of (1*) Mary could harbor the belief of another individual that that individual is both my friend and a bus driver. Now, if this individual is not my friend, or not a bus driver, we would have the divergence of truth conditions for (3a) and (1*). Alternatively, we could have a particular individual which is a friend of mine, that Mary believes is a bus driver but does not believe is my friend, which would disconnect the truth conditions of (3b) and (1*).

Following Quine (1956), we can call the (2a) readings *de dicto* and the (2b) readings *de re*, but where Quine thought it was incoherent to quantify into intensional contexts such as that of (3b) and (1*)[15], there seem to be cases where this is warranted. Consider the following:[16]

(4) Ralph believes that the president of Russia is bald.

Now, suppose that Ralph holds the further belief that Putin is president, but we all know that Medvedev is president. How do we interpret (4)? Does Ralph believe that Putin is bald or that Medvedev is bald? Again, we have an ambiguity. On one interpretation, where the definite description takes scope under the propositional attitude report, we get the reading from Ralph's perspective, that Putin is bald:

(5a) Ralph believes $(\imath x)$ (x is a president of Russia and x is bald)

But when the definite description takes scope over the propositional attitude report, we get an interpretation from our perspective, where the description is used to denote the actual president, Medvedev:

(5b) $(\imath x)$ (x is a president of Russia and Ralph believes that x is bald)

Thus, it would seem that the *de re* reading is not opaque when interpreted from our perspective rather than the perspective of that to which we are attributing a belief.

In Quine's terminology, (5a) corresponds to the *de dicto* reading and (5b) corresponding to the *de re* reading. Notice, however, that in this case the intermediate reading forces a reading from Ralph's perspective:

(4*) $(\imath x)$ Ralph believes (x is a president of Russia and x is bald)

[15] Quine (1953)
[16] Aloni, Maria (2001), p. 51

Due to the attitude verb falling under the scope of the quantifier, a singular thought is required, but since both predications fall under the scope of the attitude verb, there is no way to shift the perspective away from Ralph. So, we have three degrees of involvement: the *de dicto* reading, where singular thought is not necessary and the context is opaque, the intermediate reading, where singular thought is necessary but the perspective of the subject is forced, and the *de re* reading, where singular thought is necessary but quantification-*in* is vindicated through adopting the perspective of the attributor.

With the preceding discussion in hand, I now show that the derivation of Theorem FP requires singular thought about a proposition unknown to the subject in question, by the subject in question. To this end, note that the result of FP is often cast in the following terms:

(FP) It is not possible to know an unknown truth.

So, it is not unsympathetic to consider the following a translation of the assumption for *reductio*:

(FP_R) S knows an unknown truth.

From (FP_R), it would seem that we could disambiguate in any of the three ways provided above:

(FP_{DD}) S knows that there exists an unknown truth.

(FP_{DR}) There is a truth such that S knows that it is unknown.

(FP_I) There is a proposition such that S knows that it is true and unknown.

Notice, however, that the (FP_I) interpretation's most natural reading is that the proposition is both true, known by S, but unknown by some other subject, S'. Thus, where S' is just some agent other than S, we have the following readings of (FP_I):

(FP'_I) There is a proposition such that S knows that it is true and $\text{unknown}_{S'}$.

(FP''_I) There is a proposition such that S knows that it is true and unknown_S.

In what follows, I show that the only relevant reading in which the *reductio* actually succeeds is the (FP''_I) reading.

To begin, take the *de dicto* reading. This reading has the attitude verb taking scope over the existential quantifier, formalized as follows:[17]

$$\exists s(sK(\exists\rho(\rho \wedge \neg sKp)))$$

[17]In what follows, I will be suppressing quantification over times for expository purposes.

Distribution is blocked on this reading unless we allow for distribution over the quantifier, with a principle such as this:

Q-JUMP. $K\exists\rho_1\exists\rho_2(\rho_1 \wedge \rho_2) \Leftrightarrow \exists\rho_1\exists\rho_2(K\rho_1 \wedge K\rho_2)$

But allowing both Q-JUMP and K-DIST collapses the distinction between the (FP_{DD}) reading and the ($\text{FP}_I^{''}$) reading, with the following result:

$$K(\exists\rho_1\exists\rho_2(\rho_1 \wedge \rho_2)) \dashv\vdash \exists\rho_1\exists\rho_2 K(\rho_1 \wedge \rho_2)$$

Now, what we have here is a collapse of the only reading in which a singular thought is not required into a reading in which it is necessary. Further, given that the examples provided earlier show that the truth conditions of the *de dicto* reading and the intermediate reading diverge, it would seem that we must reject either Q-JUMP or K-DIST. And since K-DIST has antecedent motivations independent from the paradox, it would seem that Q-JUMP has to go. Without Q-JUMP, KIT would have to be applied prior to the $\exists E$ subproof, and no contradiction is derivable from $p \wedge \neg Kp$.

The lack of contradiction in the strict *de re* reading is more straightforward, as the relevant formalization would just be:

$$\exists s\exists\rho(\rho \wedge sK\neg sK\rho)$$

From this reading, all we are entitled to say is that from the point of evaluation, some proposition is true and S knows that she does not know it. This is somewhat similar to the ($\text{FP}_I^{'}$) reading, which states that there is some truth that S knows that some $S' \neq S$ does not know:

$$\exists s\exists s'\exists\rho(s \neq s' \wedge sK(\rho \wedge \neg s'K\rho))$$

Again, no contradiction is derivable, the subjects differ. In the end we just get an apparent contradiction, which states that one subject knows something another does not:

$$sKp \wedge \neg s'Kp$$

Thus, the only relevant reading with a derivable contradiction is the type two intermediate reading. But note that this reading requires singular thought on the part of the subject about the proposition in question, as the perspective of the subject is forced.

Recalling the target sentence (FP_I''), the quantification involved is second-order; we are quantifying over and predicating truth of propositions. Hence,

it would seem that what is required is a singular thought about an abstract object, the proposition of which we are predicating truth. The attitude report in question here is a relation between the subject and both the proposition and a second clause, that the subject does not stand in said relation to the proposition. On its face, this looks like an outright contradiction, but before moving on, there are some peculiar features that should be pointed out. First, since the relation between the subject and the proposition in the first conjunct is a predication of truth, it could be made to seem somewhat less strange if we considered it a case of semantic ascent. To be as neutral as possible on the structure of propositional content[18], we could just say that S stands in the K relation to an ordered n-tuple of properties or predicates $\langle P_1, \ldots, P_n \rangle$ and an ordered n-tuple of entities $\langle e_1, \ldots, e_n \rangle$ such that each k-th term entity satisfies the k-th term predicate. What generates the contradiction here is that S stands in this relation to the n-tuples, but not to their content.

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[18] For an appealing account, Cf. Larson and Ludlow (1993). For a more recent account, Cf. King (2007).

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